CSC236 A2

1.

(a)

The number of elements of T that have

0 left parentheses: 1

\*

1 left parentheses: 1

(\*\*)

2 left parentheses: 2

(\*(\*\*)), ((\*\*)\*)

3 left parentheses: 5

((\*(\*\*))\*), (((\*\*)\*)\*),

((\*\*)(\*\*)),

(\*(\*(\*\*))), (\*((\*\*)\*))

4 left parentheses: 14

(((\*(\*\*))\*)\*), ((((\*\*)\*)\*)\*), (((\*\*)(\*\*))\*), ((\*(\*(\*\*)))\*), ((\*((\*\*)\*))\*),

((\*(\*\*))(\*\*)), (((\*\*)\*)(\*\*)),

((\*\*)(\*(\*\*))), ((\*\*)((\*\*)\*)),

(\*((\*(\*\*))\*)), (\*(((\*\*)\*)\*)), (\*((\*\*)(\*\*))), (\*(\*(\*(\*\*)))), (\*(\*((\*\*)\*)))

(b)

Let .

Define the number of different elements of T with n left parentheses.

Then

Explanation:

Let .

Let be the number of left parentheses has, respectively.

Let .

Then is an element of T with n left parentheses. (The number of left parentheses of ( adds one more left parenthesis)).

Thus the number of all possible permutations of with left parentheses (order matters).

Since and , we know: and

Then = the number of all possible permutations of with

left parentheses, where .

= , where .

=

# since the order matters, i.e., and

are different elements of T.

=

=

Therefore, I’ve explained why

3.

(a)

Proof:

Define

I will prove that using complete induction.

I will assume, without proof, that for any integer . (by lemma 3.3 from the course notes)

Inductive step:

Let , assume .

I will show that follows.

Base case:

, which is no smaller than the value of T on any smaller positive

natural number, since there are no smaller natural numbers. So holds.

, so holds since 1 is the only positive natural number less than 2.

Case :

Since .

So holds (by inductive hypothesis) and (by transitivity of ) we need only show that .

Since we have:

# by definition of T, since

# by since

From the above, I’ve proven that .

Therefore, T is nondecreasing.

(b)

Proof:

Define

Note that when and , this is equivalent to

I will prove that using simple induction on .

Base case:

So holds

Inductive step:

Let

Assume , that is .

I will show that follows, that is .

Since , so by definition of T:

# by definition of T, since

# since is an integer

# by inductive hypothesis,

holds, that is

From the above, I’ve proven that .

Therefore, .

(c)

Proof:

Wants to show:

Define

Then

From part (a) we know: T is nondecreasing

From part (b) we know:

Show that :

Let . Then .

Let . Then .

Let n be an arbitrary natural number no smaller than B.

Then

# since T is nondecreasing and

# since

#

#

# since

Since is the time cost for some operations, must be a constant no smaller than 0.

Therefore, since differs from by adding a constant .

Show that :

Let . Then .

Let . Then .

Let n be an arbitrary natural number no smaller than B.

Then

# since T is nondecreasing and

# since

#

#

# since

Since is the time cost for some operations, must be a constant no smaller than 0.

Therefore, since differs from by adding a constant .

From the above, I’ve proven that and .

Therefore, .